

# Small- $x$ Resummation Effects in Electroweak Processes

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## Abstract

We investigate small- $x$  resummation effects in QCD coefficient functions for  $Z_0g$  and  $Wg$  fusion processes, and we compare them with the known ones of  $\gamma g$  type. We find a strong process dependence, that we argue to be due to the possible presence of collinear singularities for either small or large  $\mathbf{k}$  of the exchanged gluon. For top quark production, we find that the  $gg \rightarrow t\bar{t}$  and  $Z_0g \rightarrow t\bar{t}$  channels have larger resummation enhancements than the  $Wg \rightarrow t\bar{t}$  one.

# 1 Introduction

In the past few years, various applications have been developed [1-5] of the  $\mathbf{k}$ -factorization method [1, 2] to combine high-energy behaviour in QCD with the renormalization group ( $RG$ ).

On one hand, for small- $x$  hard processes of strongly interacting partons, resummation formulae have been derived for various coefficient functions [1] and for the next-to-leading quark entries [3] of the singlet anomalous dimension matrix. Resummation effects turn out to enhance scaling violations in the HERA energy range [6, 7], and could be responsible for the small- $x$  rise of structure functions seen experimentally [8].

On the other hand,  $\mathbf{k}$ -factorization is also relevant for the spontaneously broken electroweak theory [4], where it provides a generalization of the effective  $W$  approximation [9] for high-energy weak boson fusion processes, like, e.g.,  $Z_0 g \rightarrow t\bar{t}$  and  $Wg \rightarrow t\bar{b}$ . In such cases, it takes into account correctly important high-energy off-shell effects which are neglected in the (naïve) collinear approximation [10].

The purpose of the present note is to calculate in detail the coefficient functions of  $Z_0 g$  and  $Wg$  fusion type by concentrating on gluon off-shell effects, and to compare the corresponding resummation formulae with the one of  $\gamma g$  type previously found. Note that while for a virtual photon source both transverse and longitudinal coefficient functions are available, here we limit ourselves to longitudinal gauge bosons, whose exchange dominates the fusion process, and is related - by the off-shell equivalence theorem [4, 11] - to the corresponding Goldstone boson exchange.

It will turn out eventually that resummation effects are strongly process dependent, and do not enhance much the  $Wg \rightarrow t\bar{b}$  channel compared to, say, the  $Z_0 g, gg \rightarrow t\bar{t}$  ones for top production. Part of the paper is devoted to explore the reasons for such a fact, and also to provide a quick way to estimate enhancement factors in the various cases.

Roughly speaking, off-shell effects yield a cross-section increase because the phase space for the exchanged gluon transverse momentum  $\mathbf{k}$  opens up at high-energies, and is furthermore weighted by the anomalous dimension exponent  $\gamma_N \simeq (3\alpha_s/\pi)(N-1)^{-1}$ ,

$N$  being the moment index. Therefore, for large enough energy  $s$ , the integration region where  $|\mathbf{k}|$  exceeds the hard scale  $Q$  ( $Q^2 \ll \mathbf{k}^2 \ll s$ ) becomes increasingly important, if not suppressed by the squared matrix element.

Let us refer to the phase space region  $Q \leq |\mathbf{k}|$  as "the disordered  $\mathbf{k}$ " region, to distinguish it from the normally ordered region  $Q_0 \leq |\mathbf{k}| \leq Q$ , typical of the incoming parton jet. We find that the size of the cross-section enhancement at high energies is related to the possible existence of a collinear singularity in the disordered  $\mathbf{k}$  region, where the exchanged quark aligns to the hard probe, rather than to the parton in the opposite direction. We also give the splitting functions of GLAP type which describe quantitatively the hard off-shell cross section, thus generalizing what already noticed in the photoproduction case [1].

For instance, while the DIS hard cross section  $\hat{\sigma}_L = \sigma(\gamma_L g \rightarrow q\bar{q})$  - due to longitudinal photons - is not particularly enhanced, the one for  $Z_L g \rightarrow t\bar{t}$  is instead normally enhanced, because of the collinear behaviour of the Goldstone boson  $\rightarrow Q\bar{Q}$  process. The corresponding splitting function yields a reasonable estimate of the enhancement factor.

On the other hand,  $\hat{\sigma}(W_L g \rightarrow t\bar{b})$  is not particularly enhanced, despite the existence of similar collinear properties of the  $W$ -Goldstone boson. But in this case, since  $m_b \ll m_t$ , the small  $\mathbf{k}$ -region is also enhanced by a  $\log(m_t/m_b)$  factor due to the customary collinear singularity in the parton jet. Thus the ratio of large  $\mathbf{k}$  to small  $\mathbf{k}$  effects becomes of order unity.

The contents of the paper are as follows. In Sec. 2 we set up the calculation of the weak boson-gluon coefficient functions on the basis of our previous treatment of  $\mathbf{k}$ -factorization in electroweak processes. The actual analytical computation is performed in Sec. 3, and in Sec. 4 we compare the present results with the ones for the photon source. Here we also discuss the role of the disordered  $\mathbf{k}$  region, and we provide the relevant splitting functions in the various cases. Our results are discussed in Sec. 5, and some analytical details are left to Appendices A and B, where we also discuss the relevant longitudinal splitting functions.

## 2 k-Factorization for V-g fusion processes

We consider here contributions to heavy quark production in which the fusion subprocess is of the mixed type, e.g.  $Z_0 g \rightarrow t\bar{t}$ ,  $W g \rightarrow t\bar{b}$ , the hard scale being essentially the top quark mass  $m_t$  (Fig.1). Since we have already discussed [4] off-shell effects in the weak-boson channel for on-shell gluons, we concentrate here on the strong interaction effects, and the corresponding resummation formulae of QCD corrections.

The fusion process that we consider (Fig. 1) is, to start with, described by double- $k_\perp$  factorization, in which the hard subprocess cross section  $\hat{\sigma}(V(q)g(k) \rightarrow Q_1\bar{Q}_2)$  is factorized from the structure functions in the hadron (lepton) at hand. This means that in the high energy regime

$$s \gg m_i^2, \mathbf{q}^2, \mathbf{k}^2, \quad \frac{q^2}{\hat{s}}, \frac{k^2}{\hat{s}} \text{ fixed}, \quad (2.1)$$

the  $Q_1\bar{Q}_2$  production cross section takes the form

$$\sigma_{Q_1\bar{Q}_2}(s, m_i^2) = \int d^2\mathbf{q} d\bar{y} d^2\mathbf{k} dz \mathcal{F}_V(\bar{y}, \mathbf{q}) \mathcal{F}_g(z, \mathbf{k}) \hat{\sigma}_{Vg}\left(\frac{\mathbf{q}}{\sqrt{\hat{s}}}, \frac{\mathbf{k}}{\sqrt{\hat{s}}}, \frac{m_i^2}{\hat{s}}\right). \quad (2.2)$$

Here  $\mathcal{F}_V$  ( $\mathcal{F}_g$ ) are the unintegrated  $V(g)$  structure functions of the initial particles, and  $\hat{\sigma}$  is defined by a projection with eikonal vertices ( appropriate in the high energy limit ) of the off-shell  $Vg \rightarrow Q_1\bar{Q}_2$  absorptive part  $A^{Vg}$ , in a physical gauge  $A \cdot n = 0$ , as follows

$$\hat{\sigma}_{Vg} = \frac{1}{2s} \frac{\bar{y}z}{q^2 k^2} \bar{p}^\mu \bar{p}^\nu p^\rho p^\sigma A_{\mu\nu, \rho\sigma}^{Vg} \quad (2.3)$$

In the following, we assume  $\mathcal{F}_V$  to be given by the lowest order V-emission probability off the external fermion, i.e.,

$$\mathcal{F}_V(\bar{y}, \mathbf{q}) = \frac{g_V^2 + g_A^2}{4\pi^3} \frac{|q^2|}{(|q^2| + M_V^2)^2} (1 - \bar{y}) \quad (2.4)$$

On the other hand, the unintegrated gluon density,  $\mathcal{F}_g(z, \mathbf{k})$  will also contain the QCD radiative corrections that we want to take into account, and is defined in terms of the gluon-hadron absorptive part in the gauge  $\bar{p} \cdot A = 0$  in the usual way [1]

$$\mathcal{F}_g(z, \mathbf{k}) = \int \frac{dk^2}{(2\pi)^4} \frac{\mathbf{k}^2}{zs^2} \bar{p}^\mu \bar{p}^\nu G_{\mu\nu}(k, p) \quad (2.5)$$

so that

$$\int_0^{Q^2} d^2k \mathcal{F}_g(z, \mathbf{k}) = g^A(z, Q^2) \quad (2.6)$$

is the corresponding gluon density in the initial hadron A.

The  $\mathbf{k}$ -factorized expression (2.2) shows off-shell effects due to the dependence of  $\hat{\sigma}$  on the transverse momenta  $\mathbf{q}(\mathbf{k})$  of the weak boson (gluon). The  $\mathbf{q}$ -dependence was investigated in Ref.[4] and amounts to calculable corrections to the effective-W approximation in the high energy limit. The effect is not too large, and was estimated to be of the order of 10% for the present value of the top mass.

On the other hand, the  $\mathbf{k}$ -dependence is enhanced for  $s \gg m_t^2$ , or  $z \ll 1$ , by large anomalous dimension effects in  $\mathcal{F}_g$ . They come from QCD perturbative contributions of the form  $z^{-1} g_n(k^2, Q_0^2) (\alpha_s \log(1/z))^n$ , which are resummed by the BFKL equation [12], and are translated in the coefficient function by the  $\mathbf{k}$ -integration in Eq.(2.2).

In order to concentrate on strong interaction effects, we shall take in Eq.(2.2) the "small  $\mathbf{q}$ " limit for the weak boson, so as to translate it into a single- $\mathbf{k}$  factorization formula. This is done by setting, for  $\mathbf{q} = O(M_V^2)$ ,

$$q^\mu \simeq \bar{y} \bar{p}^\mu + \mathbf{q}^\mu \rightarrow \bar{y} \bar{p}^\mu \quad (2.7)$$

and by using in Eq.(2.3) the Ward identity [4, 11]

$$\bar{y}^2 \bar{p}^\mu \bar{p}^\nu A_{\mu\nu, \rho\sigma}^{Vg} \simeq q^\mu q^\nu A_{\mu\nu, \rho\sigma}^{Vg} = M_V^2 A_{\rho\sigma}^{Gg}, \quad (2.8)$$

which relates the longitudinal V amplitudes to the ones for the corresponding Goldstone bosons, and yields

$$|q^2| \hat{\sigma}_{Vg}|_{\mathbf{q}^2=0} = \frac{M_V^2 z}{2\bar{y}s} \frac{p^\rho p^\sigma}{|\mathbf{k}^2|} A_{\rho\sigma}^{Gg} \equiv \hat{\sigma}_{Gg} \left( \frac{|k^2|}{z\bar{y}s}, \frac{m_i^2}{|k^2|} \right) \quad (2.9)$$

We then notice that the  $\mathbf{q}^2$  integration in Eq.(2.2) can be explicitly done around  $|q| \simeq M_V$  by using Eq.(2.8) and the expression (2.4) for  $\mathcal{F}_V$ , to obtain

$$\sigma_{Q_1 \bar{Q}_2}(s, m_i^2) = \frac{g_V^2 + g_A^2}{4\pi^2} \frac{1}{M_V^2} \int \frac{d\bar{y}}{\bar{y}} \frac{dz}{z} d^2\mathbf{k} \mathcal{F}_g(z, \mathbf{k}) \hat{\sigma}_{Gg} \left( \frac{\mathbf{k}^2}{z\bar{y}s}, \frac{m_i^2}{\mathbf{k}^2} \right) + O\left(\frac{1}{m_t^2}\right) \quad (2.10)$$

We have thus exhibited the  $O(1/M_V^2)$  part of the cross section (2.2), due to the longitudinal V polarization, that will be considered in the following, and is also dominant in the region  $|q^2| = O(M_V^2)$ , provided  $M_V^2 \ll m_t^2$ . The additional  $O(1/m_t^2)$  terms can in principle be fully evaluated from (2.2) on the basis of the perturbative analysis of the full double- $k_\perp$  cross section (2.3), but are known not to provide large off-shell effects [4] and will no longer be considered in the following.

The single- $\mathbf{k}$  factorized formula (2.10) can be further elaborated to provide a resummation formula for the corresponding coefficient function.

Firstly we define the Mellin transform in the energy variable

$$\sigma_N^{Q_1\bar{Q}_2}(m_i^2) \equiv \int_0^\infty \frac{ds}{s} \left[ \frac{s}{(m_1 + m_2)^2} \right]^{-N} \hat{\sigma}_{Q_1\bar{Q}_2}(s, m_i^2), \quad (2.11)$$

to rewrite Eq.(2.10) in the form

$$\sigma_N^{Q_1\bar{Q}_2}(m_i^2) = \frac{g_V^2 + g_A^2}{4\pi^2} \frac{1}{N-1} \frac{1}{M_V^2} \int d^2k \mathcal{F}_N(k) \hat{\sigma}_N^{Gg}\left(\frac{\mathbf{k}^2}{M^2}, \frac{m_1}{m_2}\right), \quad (2.12)$$

where the  $1/(N-1)$  factor comes from the  $\bar{y}$  integration, and we have defined  $M = m_1 + m_2$ .

Secondly, we express the  $\mathbf{k}$ -dependence of  $\mathcal{F}_N$  in terms of the BFKL anomalous dimension [10]

$$\gamma_N(\alpha_s) = \gamma_N\left(\frac{\bar{\alpha}_s}{N-1}\right) = \frac{\bar{\alpha}_s}{N-1} + 2\zeta(3)\left(\frac{\bar{\alpha}_s}{N-1}\right)^4 + \dots, \left(\bar{\alpha}_s = \frac{3\alpha_s}{\pi}\right), \quad (2.13)$$

as follows

$$\mathcal{F}_N^A(\mathbf{k}) = \frac{1}{\pi \mathbf{k}^2} \gamma_N(\alpha_s) \left(\frac{\mathbf{k}^2}{\mu^2}\right)^{\gamma_N(\alpha_s)} g_N^A(\mu^2), \quad (2.14)$$

where  $\mu = O(M^2)$  is the factorization scale.

Finally, inserting Eq.(2.14) into Eq.(2.12) allows performing the  $\mathbf{k}$ -integration in terms of the calculable  $\mathbf{k}^2$ -moments

$$h_N^{Q_1\bar{Q}_2}(\gamma) = \gamma \int \frac{d\mathbf{k}^2}{\mathbf{k}^2} \left(\frac{\mathbf{k}^2}{M^2}\right)^\gamma \hat{\sigma}_N^{Gg}\left(\frac{\mathbf{k}^2}{M^2}, \frac{m_1}{m_2}\right), \quad (2.15)$$

and provides the final result

$$\hat{\sigma}_N^{Q_1\bar{Q}_2}(m_i^2) = \frac{1}{M_V^2} \left( \frac{g_V^2 + g_A^2}{4\pi} \right) \frac{1}{\pi(N-1)} h_N^{Gg}(\gamma_N(\alpha_s(M^2))) \cdot g_N^A(M^2). \quad (2.16)$$

Here the factor in front of  $g_N^A$  provides the QCD coefficient function for the given process. Since  $\gamma_N$  in Eq(2.13) is a known function of the effective coupling  $\alpha_s/(N-1)$ , the expression (2.15), evaluated at  $\gamma_N(\alpha_s(M^2))$ , provides the resummation formula we are looking for, once the lowest order expression for the off-shell cross section  $\hat{\sigma}^{Gg}$  is given.

Note that, in deriving Eq.(2.16), we have kept, for simplicity,  $\alpha_s$  frozen at its value  $\alpha_s(M^2)$ , and we have also used the expression (2.14) even for  $\mathbf{k}^2 < Q_0^2$ , where  $Q_0$  is a scale defining the boundary of the perturbation approach ( $\alpha_s(Q_0^2) \leq 1$ ). It can be proven [1, 13] however, that using a RG improved expression (2.14) and/or a full solution of the BFKL equation [12] for  $\mathcal{F}_N(k)$ , including higher twists and possibly running coupling [14], does not change the final result (2.16), except for subleading terms of relative order  $\alpha_s(M^2)$ , which are not considered here.

Thus our procedure will be to first evaluate the perturbative expression for the off-shell  $\hat{\sigma}^{Vg}$  and then the corresponding  $h$ -function, providing the resummed coefficient.

### 3 Evaluating the coefficient function

The hard sub-process under study –  $V_L g \rightarrow Q_1 \bar{Q}_2$  – is a variant of the customary DIS process and has the simplifying feature of having only abelian-type diagrams (Fig. 2). While the corresponding matrix element is thus rather straightforward, the analytical final state integrations are non trivial due to the unequal mass kinematics and will be sketched in the following.

In order to calculate  $\hat{\sigma}^{Gg}$  in Eq. (2.9) it is convenient to introduce the off-shell transverse gluon polarizations

$$\varepsilon_{IN}^\mu = \frac{1}{\mathbf{k}} \left( \mathbf{k}^\mu + \frac{\mathbf{k}^2}{p \cdot k} p^\mu \right), \quad \varepsilon_{OUT}^\mu = (0, \boldsymbol{\varepsilon}, 0), \quad (3.1)$$

satisfying  $\varepsilon \cdot p = \varepsilon \cdot \mathbf{k} = 0$ , and the longitudinal one

$$\varepsilon_L^\mu(k) = \frac{1}{k^2} \left( k^\mu - \frac{k^2}{p \cdot k} p^\mu \right), \quad (3.2)$$

satisfying  $\varepsilon_\alpha \cdot k = 0$ , and

$$\varepsilon_\alpha \cdot \varepsilon_\beta = \eta_{\alpha\beta}, \quad \sum_\alpha \eta_\alpha \varepsilon_\alpha^\mu \varepsilon_\beta^\nu = g_{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \quad (3.3)$$

with  $\eta_{IN} = \eta_{OUT} = -\eta_L = -1$ . By using the Sudakov parametrization

$$k^\mu = z p^\mu - \bar{z} \bar{p}^\mu + \mathbf{k}^\mu = z p^\mu + \mathbf{k}^\mu + O\left(\frac{\mathbf{k}^2}{\sqrt{s}}\right); \quad k^2 = -\frac{\mathbf{k}^2}{1-z} \simeq -\mathbf{k}^2 \quad (3.4)$$

it is easy to realize that, in the high energy limit,

$$\varepsilon_L^\mu - \varepsilon_{IN}^\mu = \frac{z}{|\mathbf{k}|} p^\mu. \quad (3.5)$$

Therefore, the eikonal projection in Eq. (2.9) becomes (with  $\nu \equiv 2q \cdot p$ )

$$\hat{\sigma}_{Gg} = \frac{M_V^2 z}{4q \cdot k} \frac{p^\rho p^\sigma}{|\mathbf{k}|^2} A_{\rho\sigma}^{Gg} = \frac{M_V^2}{2z\nu} (A_{LL} + A_{IN,IN}) \quad (3.6)$$

after noticing that  $A_{L,IN} = A_{IN,L} = 0$  by the usual invariant decomposition [1, 4]

The absorptive parts occurring in the r.h.s of Eq. (3.6) are in turn obtained from the amplitudes  $M(G(q) + g(k) \rightarrow Q_1(P_1, \sigma_1) + \bar{Q}_2(P_2, \sigma_2))$  by the customary integrations over phase space and spins

$$A_{LL} + A_{IN,IN} = \sum_{\sigma_1, \sigma_2} \int d\Phi (|M_L|^2 + |M_{IN}|^2). \quad (3.7)$$

The squared amplitudes in Fig. 2, summed over spins, turn out to be

$$\begin{aligned} & \frac{M_V^2}{2z\nu} \sum_{\sigma_1, \sigma_2} (|M_L|^2 + |M_{IN}|^2) = \\ & = g_s^2 (C_A^2 (m_1 + m_2)^2 + C_V^2 (m_1 - m_2)^2) \left( \frac{z\nu}{(m_2^2 - \hat{t})(m_1^2 - \hat{u})} \right) + \end{aligned}$$



$$\begin{aligned}
& +g_s^2(C_V^2 + C_A^2) \frac{(m_1^2 - m_2^2)^2}{z\nu} \left[ \frac{2m_1^2}{(m_1^2 - \hat{u})^2} + \frac{2m_2^2}{(m_2^2 - \hat{t})^2} + \right. \\
& \quad \left. \left( \frac{z\nu}{(m_2^2 - \hat{t})(m_1^2 - \hat{u})} \right) \left( -2 + 2\frac{m_1^2 + m_2^2}{z\nu} \right) \right] + \\
& +g_s^2(C_V^2 + C_A^2) \frac{(m_1^2 - m_2^2)^2 \mathbf{k}^2}{(z\nu)^2} \left[ 4 \left( \frac{z\nu}{(m_2^2 - \hat{t})(m_1^2 - \hat{u})} \right) \left( 1 - 3\frac{(m_1^2 - m_2^2)^2}{(z\nu)^2} \right) + \right. \\
& - \left( \frac{1}{(m_2^2 - \hat{t})^2} + \frac{1}{(m_1^2 - \hat{u})^2} \right) \left( z\nu + 6\frac{(m_1^2 - m_2^2)^2}{z\nu} \right) + \\
& \quad \left. 6(m_1^2 - m_2^2) \left( \frac{1}{(m_2^2 - \hat{t})^2} - \frac{1}{(m_1^2 - \hat{u})^2} \right) \right] \tag{3.8}
\end{aligned}$$

where  $C_V(C_A)$  denote the vector (axial)  $V \rightarrow Q_1 \bar{Q}_2$  coupling constants, and we have defined  $\nu = 2q \cdot p$  and the Mandelstam variables of the hard subprocess

$$\hat{s} = (k + q)^2, \quad \hat{t} = (q - P_1)^2 = (k - P_2)^2, \quad \hat{u} = (q - P_2)^2 = (k - P_1)^2. \tag{3.9}$$

In the particular case of  $Z_0 g \rightarrow t\bar{t}$ , the heavy quark masses are equal ( $m_1 = m_2 = m$ ) and all terms in Eq. (3.8) drop out but the first, which yields

$$\frac{1}{2z\nu} |\overline{M_Z^2}| = \frac{g_s^2}{M_V^2} 4C_A^2 m^2 \left( \frac{1}{m^2 - \hat{t}} + \frac{1}{m^2 - \hat{u}} \right). \tag{3.10}$$

Therefore, there is no explicit  $\mathbf{k}$ -dependence of  $\overline{M^2}$  in this simple case.

On the other hand, in the case of  $Wg \rightarrow t\bar{b}$ , which has larger cross section for top production [15], all the terms in Eq. (3.8) contribute (with  $m_1 = m_t$  and  $m_2 = m_b$ , say) and the  $\mathbf{k}$ -dependence is more involved.

In order to deal with the general mass configuration, it is convenient to write the two body phase space in terms of rescaled invariants, as follows (Appendix A)

$$\begin{aligned}
d\Phi(1, 2) &= \frac{d^4\Delta}{(2\pi)^2} \delta^+((q + \Delta)^2 - m_1^2) \delta^+((k - \Delta)^2 - m_2^2) = \\
&= \frac{d\tau}{8\pi} \Theta(\tau) \Theta(1 - \tau) \Theta\left(z\nu - \mathbf{k}^2 - \frac{m_1^2}{\tau} - \frac{m_2^2}{1 - \tau}\right) \tag{3.11}
\end{aligned}$$

where we have performed a trivial azimuthal integration, and we have defined

$$\tau = \frac{m_1^2 - \hat{t}}{z\nu}, \quad 1 - \tau = \frac{m_2^2 - \hat{u}}{z\nu}. \tag{3.12}$$

It is also helpful to introduce the energy type variable

$$\begin{aligned}\sigma &= \tau(1 - \tau)z\nu - m_1^2(1 - \tau) - m_2^2\tau \geq \frac{1}{4}\xi\mathbf{k}^2, \\ \xi &= 4\tau(1 - \tau)\end{aligned}\tag{3.13}$$

in terms of which the phase-space boundary takes a mass-independent form:

$$d\Phi(1, 2) = \frac{d\xi}{16\pi\sqrt{1-\xi}}\Theta(\xi)\Theta(1-\xi)\Theta(4\sigma - \xi\mathbf{k}^2).\tag{3.14}$$

It turns out that, by eliminating  $z\nu$ ,  $\hat{t}$  and  $\hat{u}$  in favour of  $\sigma$  and  $\xi$ , Eq. (3.8) takes, after some algebra, a particularly simple form

$$\begin{aligned}\frac{M_V^2}{2z\nu}\overline{M^2} &= \frac{z\nu\xi}{4}g_s^2\left[(C_A^2(m_1 + m_2)^2 + C_V^2(m_1 - m_2)^2)\frac{1}{(\sigma + m_1^2)(\sigma + m_2^2)} + \right. \\ &\quad \left. + (C_V^2 + C_A^2)\frac{\xi}{4}(m_1^2 - m_2^2)^2\frac{(-2\sigma + \mathbf{k}^2(\frac{3}{2}\xi - 1))}{(\sigma + m_1^2)^2(\sigma + m_2^2)^2}\right].\end{aligned}\tag{3.15}$$

This expression has the remarkable property that the  $\sigma$ -dependence essentially factorizes from the  $\xi$ -dependence, so that the various integrations needed are decoupled, eventually.

For instance, the  $N = 1$  moment of  $\hat{\sigma}_{Gg}$ , relevant for the high energy limit of Eq. (2.12) becomes

$$\begin{aligned}\hat{\sigma}_{N=1}^{Gg}\left(\frac{\mathbf{k}^2}{M^2}, \frac{m_1}{m_2}\right) &= \\ &= g_s^2 \int_0^1 \frac{d\xi}{16\pi\sqrt{1-\xi}} \int_{\frac{\xi\mathbf{k}^2}{4}}^\infty d\sigma \left[ \frac{C_A^2(m_1 + m_2)^2 + C_V^2(m_1 - m_2)^2}{(\sigma + m_1^2)(\sigma + m_2^2)} + \right. \\ &\quad \left. + (C_V^2 + C_A^2)\frac{\xi}{4}(m_1^2 - m_2^2)^2\frac{\mathbf{k}^2(\frac{3}{2}\xi - 1) - 2\sigma}{(\sigma + m_1^2)^2(\sigma + m_2^2)^2} \right].\end{aligned}\tag{3.16}$$

The corresponding  $\mathbf{k}^2$ -moment, i.e., the  $h$ -function in Eq. (2.15) is easily performed by doing the (linear)  $\mathbf{k}^2$ -integration before the  $\sigma$  and  $\xi$ -integrations, with the result:

$$\begin{aligned}h_{N=1}^{Gg}\left(\gamma, \frac{m_2}{m_1}\right) &= \frac{2\alpha_s}{4}\left(\frac{m_1 + m_2}{2}\right)^{-2\gamma}\frac{1}{\gamma}B(1/2, 1 - \gamma)B(1 - \gamma, 1 + \gamma) \times \\ &\times \left[ (C_A^2(m_1 + m_2)^2 + C_V^2(m_1 - m_2)^2)\frac{m_1^{2\gamma} - m_2^{2\gamma}}{m_1^2 - m_2^2} + \right. \\ &\quad \left. + \frac{C_V^2 + C_A^2}{3 - 2\gamma}\left(m_1^{2\gamma} + m_2^{2\gamma} - \frac{2}{1 + \gamma}\frac{m_1^{2(\gamma+1)} - m_2^{2(\gamma+1)}}{m_1^2 - m_2^2}\right) \right].\end{aligned}\tag{3.17}$$

The equal mass case, relevant for  $Z_0 g \rightarrow Q\bar{Q}$ , is again particularly simple because only the first term in square brackets contributes. We obtain the mass-independent expression

$$h_{N=1}^{Gg}(\gamma, 1) = \frac{\alpha_s T_R}{4\pi} \pi C_A^2 \frac{\Gamma(1-\gamma)^2 \Gamma(1+\gamma) \sqrt{\pi}}{\Gamma(3/2-\gamma)}. \quad (3.18)$$

Of particular interest for estimating the size of scaling violations is the behaviour of Eq. (3.16) around  $\gamma = 0$  (collinear limit) and  $\gamma = 1/2$  (saturating value [16] of the BFKL anomalous dimension). The  $\gamma = 0$  limit is strongly dependent on the mass ratio  $r = m_2/m_1$  and has the form

$$h_{N=1}^{Gg}(0, r) = \frac{\alpha_s T_R}{2} \left[ \frac{2}{3} (C_V^2 + C_A^2) \left( 2 \frac{1+r^2}{1-r^2} \log \frac{1}{r} \right) - \frac{4r}{1-r^2} (C_V^2 - C_A^2) \log \frac{1}{r} \right]. \quad (3.19)$$

On the other hand, the large  $\gamma$  behaviour is roughly determined by the presence of a  $\gamma = 1$  double pole, with behaviour

$$h_{N=1}^{Gg}(\gamma, r) \underset{\gamma \rightarrow 1}{\simeq} \frac{\alpha_s T_R}{\pi} \left[ \pi \left( C_A^2 + \left( \frac{1-r}{1+r} \right)^2 C_V^2 \right) \frac{1}{(1-\gamma)^2} \right], \quad (3.20)$$

to which, in Eq. (3.17), only the first term in square brackets contributes. The saturating value at  $\gamma = 1/2$  is instead given by

$$h_{N=1}^{Gg}(1/2, r) = \frac{\pi^2}{2} \alpha_s \left[ C_A^2 + \left( \frac{1-r}{1+r} \right)^2 \left( C_V^2 - \frac{1}{6} (C_V^2 + C_A^2) \right) \right], \quad (3.21)$$

to be compared with the rough estimate obtained by setting  $\gamma = 1/2$  in Eq. (3.20).

The overall behaviour of  $h^{Gg}(\gamma)$  is plotted in Fig. 3 for various mass ratios. In the equal mass case it increases from the  $\gamma = 0$  limit all the way up to the double pole at  $\gamma = 1$ . The enhancement ratio at  $\gamma = 1/2$  (the asymptotic value at the BFKL Pomeron singularity) is

$$\left. \frac{h^{Gg}(1/2)}{h^{Gg}(0)} \right|_{r=1} = \left( \frac{\pi}{2} \right)^2 \simeq 2.47, \quad (3.22)$$

a value typical of other single- $\mathbf{k}$  processes, like heavy flavour photoproduction or DIS scaling violations [1, 3].

Thus we conclude that the  $gZ \rightarrow Q\bar{Q}$  fusion process has rather large resummation effects, while being disfavoured at Born level with respect to the  $gW$  and  $gg$  fusion channels for top production.

On the other hand the unequal mass  $gW \rightarrow t\bar{b}$  case is peculiar because the  $\gamma = 0$  coefficient (Born cross section) is enhanced, according to Eq. (3.19), by a large factor  $\sim \log m_t/m_b$  due, as we shall see in Sec. 4, to a collinear singularity in the  $m_b = 0$  limit. Thus the  $h$ -function starts decreasing away from  $\gamma = 0$ , has a minimum, and then is driven up to larger values by the  $\gamma = 1$  double pole (Fig. 3b).

As a consequence, the enhancement ratio

$$\left. \frac{h^{Gg}(1/2)}{h^{Gg}(0)} \right|_{r \ll 1} = \left( \frac{\pi}{2} \right)^2 \frac{5}{\log \frac{1}{r} + 1} \simeq 1.49, \quad \left( r \approx \frac{1}{30} \right), \quad (3.23)$$

is not large, and finite energy resummation effects are not likely to be important.

The above discussion shows that the relative importance of the  $Z_0g$  vs  $Wg$  channel for top production is energy dependent, the latter being dominant at low energies, but less enhanced by resummation effects. In any case, the most important process for top production is expected to be gluon-gluon fusion which yields a greater cross section at low energies and is expected to be strongly enhanced by QCD resummation effects [1] at energies of LHC type (Cf. Sec. 4C).

## 4 Process dependence of resummation effects

We have already noticed that resummation effects due to large small- $x$  anomalous dimensions are substantially different in the  $Z_0g \rightarrow t\bar{t}$  vs  $Wg \rightarrow t\bar{b}$  cases. Comparing with the known coefficient functions of (heavy) quark production with transverse [1] and longitudinal [3] photon sources, we find the enhancement ratios  $h(1/2)/h(0)$  listed in Table 1.

We can thus roughly distinguish "normally enhanced" processes ( $Z_L g$ ,  $\gamma_T g$ ) from not enhanced ones ( $W_L g$ ,  $\gamma_L g$ ). We would like to understand this fact on the basis of the observation [1] that collinear singularities in the internal momenta may enhance the  $\mathbf{k}^2$ -moments of the hard probe cross section.

Since we compare large  $\gamma$  with small  $\gamma$  moment indices, we should distinguish at least two cases.

### A) Small $\mathbf{k}$ enhancement.

This occurs in some massless limit, e.g.  $m_q \rightarrow 0$  in DIS-type processes, or  $m_b \ll m_t$  in the cases examined here. The collinear singularity is due to a (nearly) massless quark exchange of virtuality  $\hat{t}$  in the region  $\mathbf{k}^2 \ll |\hat{t}| \ll Q^2$ , where  $Q$  (or  $M$ ) is the hard scale. It yields the  $\log m_t/m_b$  factor in Eqs. (3.19) and (3.23), and a  $\log Q^2/\mathbf{k}^2$  factor in the DIS case, where it provides a  $1/\gamma$  pole of the coefficient function  $h_2(\gamma)$  (Cf. Ref.[3]). This enhancement, due to a "normal" collinear behaviour in the target jet, is important in order to assess the magnitude of the nearly on-shell (or  $\gamma = 0$ ) region.

### B) Disordered- $\mathbf{k}$ enhancements

This is the typical off-shell effect that we are exploring in the high-energy regime, in which the  $\mathbf{k}$ -phase space opens up away from the normal collinear region mentioned before.

We shall call "disordered  $\mathbf{k}$ " region the one in which  $Q^2(M^2) \ll |\hat{t}| \ll \mathbf{k}^2 < z\nu$  and therefore the exchanged quark aligns to the electroweak boson rather than to the incoming hadron. In such region the off shell cross section  $\hat{\sigma}_{Hg}$  for a hard source ( $V$  or  $\gamma$ ) coupled to quarks, is again dominated by a collinear singularity as follows

$$\hat{\sigma}_{Hg}\left(\frac{Q^2}{\mathbf{k}^2}, \frac{Q^2}{z\nu}\right) = g_H^2 \frac{Q^2}{z\nu} P_{H \rightarrow q\bar{q}}\left(\frac{\mathbf{k}^2}{z\nu}\right) \frac{\alpha_s T_R}{\pi} \log \frac{\mathbf{k}^2}{Q^2}, \quad (\mathbf{k}^2 \gg Q^2). \quad (4.1)$$

Here, however,  $\bar{\tau} \equiv \mathbf{k}^2/z\nu$  is the Bjorken variable of the exchanged quark, *as probed by the hard gluon*, and  $P_{H \rightarrow q\bar{q}}$  is the corresponding splitting function.

The expression (4.1) provides the correct double-pole residue at  $\gamma = 1$  of the various coefficient functions in Table 1, where also the corresponding definition of the coupling  $g_H^2$  is quoted. In fact, the  $N = 1$  moment of (4.1) is

$$\hat{\sigma}_{N=1}\left(\frac{Q^2}{\mathbf{k}^2}\right) = \frac{\alpha_s T_R}{\pi} g_H^2 A_H \frac{Q^2}{\mathbf{k}^2} \log \frac{\mathbf{k}^2}{Q^2}, \quad (\mathbf{k}^2 \gg Q^2), \quad (4.2)$$

$$A_H \equiv \int_0^1 d\bar{\tau} P_{H \rightarrow q\bar{q}}(\tau), \quad (4.3)$$

and thus its contribution to the  $h$ -function (2.14) becomes

$$h_{N=1}^{Hg}(\gamma) \underset{\gamma \rightarrow 1}{\simeq} \frac{\alpha_s T_R}{\pi} \frac{g_H^2 A_H}{(1 - \gamma)^2}, \quad (4.4)$$

in agreement with the properly normalized expressions quoted in Table 1.

Here we notice that the source dependence comes from both the coupling constant  $g_H^2$  and  $A_H$ , the  $N = 1$  moment of the splitting function. It is easy to realize (Appendix B) that for transverse photon coefficient functions we have (Table 2)

$$P_{\gamma T \rightarrow q\bar{q}}(\bar{\tau}) = \bar{\tau}^2 + (1 - \bar{\tau})^2 \quad (4.5)$$

while for longitudinal weak bosons (Goldstone bosons) we have

$$P_{G \rightarrow q\bar{q}}(\bar{\tau}) = 1, \quad (4.6)$$

as also quoted in Table 2.

It is clear that the double pole approximation provides most of the enhancement factor only in those cases in which the  $\gamma = 0$  region is not enhanced. This explains the "normal" enhancement of  $\gamma_T g$  and  $Z_L g$  processes and the small enhancement of  $W_L g$  processes.

Particular attention is needed for the  $\gamma_L g$  process, occurring in the DIS longitudinal structure function, which is substantially different from the corresponding case in broken gauge theories. In fact, there is no Goldstone boson in this case, and the region (A) is not collinear singular due to the helicity flip zero at  $q^2 = 0$  for Breit frame scattering

of massless spin 1/2 quarks. However, the disordered region (B) happens to have a linear collinear divergence, being essentially of Coulomb scattering type (Appendix B). Its contribution to  $\sigma_L$  (defined by saturating with  $\varepsilon_L$  in Eq. (B.5)) is of type

$$\begin{aligned}\hat{\sigma}_{\gamma_L g} &= \frac{\alpha\alpha_s T_R N_f}{\pi} \frac{(Q^2)^2}{z\nu} \int_{\bar{\tau}Q^2}^{\hat{k}^2} \frac{d\hat{t}}{\hat{t}^2} 4\bar{\tau}^2(1-\bar{\tau}) = \\ &= \frac{\alpha\alpha_s T_R N_f}{\pi} \frac{Q^2}{z\nu} 4\bar{\tau}(1-\bar{\tau}),\end{aligned}\quad \bar{\tau} = \frac{\mathbf{k}^2}{z\nu},\quad (4.7)$$

and provides a *single pole* behaviour of the  $h$ -function

$$h_{\gamma_L g}(\gamma) \underset{\gamma \rightarrow 1}{\simeq} \frac{\alpha_s T_R N_f}{\pi} \frac{\pi e^2 A_L}{1-\gamma} \quad (4.8)$$

Once again, the collinear analysis sketched above explains the weak enhancement in this case, even if in a slightly more involved way.

### C) $s$ -Channel enhancement

Finally we should recall [1], for completeness, that non abelian gluon-gluon fusion processes show a third collinear region, the one in the  $s$ -channel, which is important to estimate the size of resummation effects, e.g. in hadroproduction of heavy flavours. This is a phase space region of the hard subprocess  $gg \rightarrow Q\bar{Q}$  in which  $M^2 \ll \hat{s} \ll z\nu$  and  $M^2 \ll \mathbf{l}^2 = (\mathbf{q} + \mathbf{k})^2 \ll \mathbf{k}^2 \simeq \mathbf{q}^2$ . So that *both*  $\mathbf{k}^2$  and  $\mathbf{q}^2$  are large and of the same order. Therefore, the massless quark limit is relevant, the cross section  $\hat{\sigma}_{gg}$  behaves as  $1/\hat{s}$ , and its first moment is dominated by the *intermediate* gluon collinear singularity

$$\hat{\sigma}_{N=1}^{gg} \left( \frac{\mathbf{l}^2}{M^2} \right) = \frac{A_H N_c \alpha_s^2}{\mathbf{l}^2} \log \frac{\mathbf{l}^2}{M^2} \quad (4.9)$$

for one produced flavour. Furthermore, this time

$$A_H = \int d\tau P_{g \rightarrow q\bar{q}}(\tau) \quad (4.10)$$

is related to the gluon  $\rightarrow$  quark pair splitting function in the final state.

It turns out [1] that, at extreme energies, in which both gluon transverse momenta  $\mathbf{q}$  and  $\mathbf{k}$  carry a large anomalous dimension  $\gamma_N(\alpha_s)$ , the behaviour (4.9) causes a *triple pole* in the coefficient function

$$C^{gg}(\alpha_s) \simeq (1 - 2\gamma_N)^{-3} A_H N_c \alpha_s^2 \quad (4.11)$$

which occurs at precisely the asymptotic value  $\gamma = 1/2$  because of the double- $k$  dependence.

Of course, this implies that the non-abelian  $s$  - *channel* region is asymptotically dominant, so that the  $gg \rightarrow Q\bar{Q}$  channel is more strongly enhanced than the  $\gamma g$  and  $Vg$  ones.

## 5 Discussion

There are various outcomes of the analysis presented in this paper. The first one is that the relative importance of the  $Wg$  vs the  $Z_0g$  fusion processes for top production changes with energy, when scaling violations drive the coefficient away from the  $\gamma = 0$  region, collinear singular for  $Wg \rightarrow t\bar{b}$  only. However, the  $gg \rightarrow t\bar{t}$  process is even more enhanced by scaling violations, and is thus confirmed as the most important at energies of LHC type.

The second point is that the enhancement factors are roughly described by the disordered- $\mathbf{k}$  collinear singularities, which are process dependent, since they involve splitting functions of the hard source (transverse or longitudinal  $\gamma$ 's,  $Z$ 's and  $W$ 's in the present case). This leads to a variety of enhancement factors, and implies that it is not really possible to eliminate *all* large resummation effects by a redefinition of the gluon density.

This observation is relevant for the analysis of small  $x$  scaling violations being performed at HERA. While it is fruitful to look for a factorization scheme -or definition of the gluon density- which incorporates some *universal* enhancement factors, one has to



live with the fact that resummation effects are sizeable for *some* processes, and may in fact be needed to explain the small  $x$  rise of structure functions.

Thus, a *comparative* study of various processes (e.g.  $F_2, F_L$ , heavy flavour production) is needed at both theoretical and experimental level.

There is a final point to be noticed. The collinear analysis performed in this paper, and compared to exact results, provides in fact a method which can be generalized to gluon-gluon kernels and other processes for which exact results are not yet available. In fact, it provides a quick way to estimating resummation effects and their process dependence, even for non abelian hard subprocesses. This analysis is left to future investigations.

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## Appendix A: Unequal mass kinematics

For the process  $G(q) + g(k) \rightarrow Q_1 \bar{Q}_2$  we use a Sudakov parametrization of momenta, which, in the notation of Fig. 1, reads

$$q^\mu = \bar{y}\bar{p}^\mu + \mathbf{q}^\mu \quad (q^2 = -Q^2), \quad (\text{A.1})$$

$$k^\mu = zp^\mu + \mathbf{k}^\mu \quad (k^2 = -\mathbf{k}^2), \quad (\text{A.2})$$

$$\Delta^\mu = (-q + P_1)^\mu = (k - P_2)^\mu = z\tau p^\mu - \bar{y}\bar{\tau}\bar{p}^\mu + \Delta^\mu \quad (\text{A.3})$$

where  $\bar{y}(z)$  is the momentum fraction of the probe (gluon) with respect to the incoming lepton momentum  $\bar{p}$  (hadron momentum  $p$ ).

Then the mass-shell conditions  $P_i^2 = m_i^2$  yield

$$\tau(1 - \bar{\tau}) = \frac{m_1^2 + (\mathbf{\Delta} + \mathbf{q})^2}{z\nu}, \quad (1 - \tau)\bar{\tau} = \frac{m_2^2 + (\mathbf{\Delta} - \mathbf{k})^2}{z\nu} \quad (\text{A.4})$$

where  $\nu = 2q \cdot p = \bar{y}s$ . Therefore, the expression of the invariants in terms of longitudinal and transverse variables is easily obtained. The  $\nu$ -phase space is described by

$$z\nu = \frac{m_1^2 + (\mathbf{\Delta} + \mathbf{q})^2}{\tau} + \frac{m_2^2 + (\mathbf{k} - \mathbf{\Delta})^2}{1 - \tau} = \frac{m_1^2 + (\mathbf{\Delta} + \mathbf{q})^2}{1 - \bar{\tau}} + \frac{m_2^2 + (\mathbf{k} - \mathbf{\Delta})^2}{\bar{\tau}} \quad (\text{A.5})$$

The first of these expressions is important for the normal collinear kinematics, the second one for the disordered- $\mathbf{k}$  region. The remaining invariants are

$$\hat{t} = (q - P_1)^2 = m_1^2 - z\nu\tau + \mathbf{q}^2 + 2\mathbf{q} \cdot \mathbf{\Delta} = -\frac{m_1^2\bar{\tau} + (\mathbf{\Delta} + \bar{\tau}\mathbf{q})^2}{1 - \bar{\tau}} - \mathbf{q}^2\bar{\tau} \quad (\text{A.6})$$

$$\hat{u} = (q - P_2)^2 = m_2^2 - z\nu(1 - \tau) + \mathbf{q}^2 + 2\mathbf{q} \cdot (\mathbf{k} - \mathbf{\Delta}) \quad (\text{A.7})$$

$$\hat{s} = z\nu - (\mathbf{k} + \mathbf{q})^2 \geq \frac{m_1^2}{\tau} + \frac{m_2^2}{1 - \tau}. \quad (\text{A.8})$$

Eqs. (A.6) and (A.7) explain the definition (3.12) of  $\tau$  (for  $\mathbf{q}^2 = 0$ ). By replacing (A.4) in the two-body phase space, and by performing the  $\bar{\tau}$  integration first, we obtain

$$d\Phi(1, 2) = \frac{d\tau}{\tau(1 - \tau)} \frac{d^2\mathbf{\Delta}}{2(2\pi)^2} \delta\left(\frac{m_1^2}{\tau} + \frac{m_2^2}{1 - \tau} + \frac{(\mathbf{\Delta} - \tau\mathbf{k})^2}{\tau(1 - \tau)} + \mathbf{k}^2 - z\nu\right) \quad (\text{A.9})$$

and by then performing the  $d^2\mathbf{\Delta}$  integration we arrive at the expression (3.11) of the text.

Finally Eq. (A.6) provides the phase space boundary

$$|\hat{t}| = |\Delta|^2 \geq \bar{\tau}\left(\frac{m_1^2}{1 - \bar{\tau}} + \mathbf{q}^2\right) \quad (\text{A.10})$$

relevant for the disordered- $\mathbf{k}$  kinematics, in which, by (A.5)  $\bar{\tau} \simeq \mathbf{k}^2/z\nu$ .

## Appendix B: Longitudinal and scalar splitting functions

Here we want to derive the splitting functions  $P_H \rightarrow q\bar{q}$  relevant for the collinear behaviour in the disordered- $\mathbf{k}$  region, because they may involve longitudinal and scalar polarizations (possibly in a broken gauge theory), which are not usually treated in the literature.

With the notation of Fig. 1 and referring to the splitting process  $V(q) \rightarrow Q_1(q + \Delta) + \bar{Q}_2(-\Delta)$ , we consider the probe momentum  $q^\mu$  to be off-shell and we use a Sudakov parametrization with massless quarks

$$q^\mu = \bar{p}^\mu - \frac{Q^2}{\nu} p^\mu, \quad (q^2 = -Q^2, \ 2p \cdot \bar{p} = \nu), \quad (\text{B.1})$$

$$-P_1^\mu = (q + \Delta)^\mu = (1 - \bar{\tau})\bar{p}^\mu + \frac{\Delta^2}{(1 - \bar{\tau})\nu} p^\mu + \Delta^\mu, \quad ((q + \Delta)^2 = 0), \quad (\text{B.2})$$

$$P'^\mu = -\Delta^\mu = \bar{\tau}\bar{p}^\mu - \frac{\Delta^2}{(1 - \bar{\tau})\nu} p^\mu - \Delta^2, \quad \left(\Delta^2 = -\frac{\Delta^2}{1 - \bar{\tau}}\right). \quad (\text{B.3})$$

where we have reabsorbed the momentum fraction  $\bar{y}$  in  $\bar{p}^\mu$ , for simplicity.

Here the relevant region of "disordered- $\mathbf{k}$ " is  $m_i^2$ ,  $Q^2 \ll \Delta^2 \ll \mathbf{k}^2$ , so that we will eventually also set  $Q^2/\mathbf{k}^2 \rightarrow 0$ .

The vector boson polarizations in the Landau gauge parallel the ones in Eqs. (3.1)-(3.3) for the gluon and are given by

$$\varepsilon_{IN}^\mu = (0, \boldsymbol{\varepsilon}_{IN}, 0), \quad \varepsilon_{OUT}^\mu = (0, \boldsymbol{\varepsilon}_{OUT}, 0) \quad (\text{B.4})$$

with  $\boldsymbol{\varepsilon}_{OUT} \cdot \boldsymbol{\Delta} = 0$ , and by

$$\varepsilon_L^\mu = \frac{1}{Q} \left( q^\mu + 2 \frac{Q^2}{\nu} p^\mu \right), \quad (\varepsilon_L^2 = 1). \quad (\text{B.5})$$

Correspondingly, the quark (antiquark) spinors with helicity  $\lambda(-\lambda')$  take the usual form

$$v^{\lambda'}(-\Delta) = \sqrt{\bar{\tau} E_q} \begin{pmatrix} \lambda' \eta^{\lambda'}(-\boldsymbol{\Delta}) \\ \eta^{\lambda'}(-\boldsymbol{\Delta}) \end{pmatrix}, \quad (\text{B.6})$$

$$u^\lambda(q + \Delta) = \sqrt{\bar{\tau} E_{q+\Delta}} \begin{pmatrix} \lambda \chi^\lambda(\Delta) \\ \chi^\lambda(\Delta) \end{pmatrix}, \quad (\text{B.7})$$

where the relevant bispinors will be approximated, in the collinear region by the expressions

$$\eta^+ = \begin{pmatrix} 1 \\ -\frac{\theta'}{2} \end{pmatrix}, \quad \eta^- = \begin{pmatrix} -\frac{\theta'}{2} \\ 1 \end{pmatrix}, \quad \theta' = -\frac{|\Delta|}{|\vec{q}|\bar{\tau}}, \quad (\text{B.8})$$

and similar ones for  $\chi^\pm$  with  $\theta = |\Delta|/|\vec{q}|\bar{\tau}$ . Note that off-shell effects are only relevant at order  $\theta^2$ .

Since we only consider, for the various initial polarizations, the leading collinear singularity, the splitting functions can be defined according to the probabilistic interpretation, by summing over quark helicities in the usual way:

$$g_H^2 P_{H \rightarrow q\bar{q}}\left(\bar{\tau}, \frac{Q^2}{\Delta^2}\right) = \frac{\bar{\tau}(1-\bar{\tau})}{2\Delta^2} \sum_{\lambda, \lambda'} |M(\varepsilon_H \rightarrow \lambda\lambda')|^2 \quad (\text{B.9})$$

## Transverse polarizations

In this well known case there is no much difference between broken and unbroken theories. The massless quark limit is smooth, the lowest order matrix elements, given in Table 3

$$M_{\lambda\lambda'} = \bar{u}^\lambda(q + \Delta)\not{\epsilon}(C_V + C_A\gamma_5)v^{\lambda'}(-\Delta) \quad (\text{B.10})$$

are helicity conserving and show the well known zero in the forward direction, due to a clash with angular momentum conservation. Correspondingly, the definition (B.9) has a finite  $Q^2 = 0$  limit, yielding the customary splitting function

$$P_{T \rightarrow q\bar{q}}(\bar{\tau}) = \bar{\tau}^2 + (1 - \bar{\tau})^2, \quad g_T^2 = C_V^2 + C_A^2, \quad (\text{B.11})$$

as quoted in Table 2. This one corresponds to the normal logarithmic collinear singularity.

## Longitudinal and scalar polarizations

Here we should single out the broken theory case, in which the  $q^\mu$  term in the longitudinal polarization (B.5) dominates the vector exchange contribution, for small  $Q^2$ , as explained in Sec. 2.

By picking up the  $q^\mu$  contribution, restoring quark masses and using the Ward Identity

$$M_{\lambda\lambda'}^L \simeq \frac{1}{Q} \bar{u}^\lambda [C_V(m_1 - m_2) + C_A(m_1 + m_2)\gamma_5] v^{\lambda'} \quad (\text{B.12})$$

we end up with effective (pseudo) scalar couplings, which are singular in the  $Q \rightarrow 0$  limit and helicity violating.

The  $1/Q$  factor in (B.12) is reabsorbed in the definition of the Goldstone boson cross section  $\hat{\sigma}_{Gg}$  in Eqs. (2.8) and (2.9), while for the (pseudo) scalar matrix element we can use the massless quark kinematics as before, thus obtaining the (pseudo) scalar values of Table 3. Note that there is again a zero in the forward direction, due to a clash with angular momentum conservation, which in this case would imply conserved helicity (!).

As a consequence, in the Goldstone case the collinear singularity is again logarithmic, with constant splitting function

$$P_{G \rightarrow q\bar{q}}(\bar{\tau}) = 1, \quad g_G^2 = C_S^2 + C_P^2 = C_A^2 \frac{(m_1 + m_2)^2 + C_V^2(m_1 - m_2)^2}{M_V^2}, \quad (\text{B.13})$$

as quoted in Table 1. Note that in the definition of  $\hat{\sigma}^{Gg}$  in Eq. (2.9) and of  $h$  in Eq. (2.16) an overall factor of  $(m_1 + m_2)^2/M_V^2$  has been taken out.

The longitudinal case in the unbroken theory is peculiar also. In fact, this time the  $q^\mu$  term in (B.5) yields vanishing contribution because of current conservation, and the  $p^\mu$  term is of order  $Q$ , but is helicity conserving. As a consequence, there is no clash with angular momentum conservation and no zero in the forward direction. By using Table 3 and Eq. (B.9) we then obtain

$$P_{L \rightarrow q\bar{q}} = 4\bar{\tau}^2(1 - \bar{\tau}) \frac{Q^2}{|\Delta|^2}, \quad |g_L|^2 = C_V^2 + C_A^2. \quad (\text{B.14})$$

As a consequence, the fact that the longitudinal process has to vanish on-shell ( $Q^2 = 0$ ), is compensated by a *linear* collinear divergence as stated in Eq. (4.6) of the text. Integration over it, taking into account the phase space boundary (A.10), provides an *effective* splitting function  $4\bar{\tau}(1 - \bar{\tau})$  but no logarithm and thus, upon  $\bar{\tau}$ -integration the same residue  $2/3$ , but a single pole, in  $h(\gamma)$ . This result is connected with the well-known fact that longitudinal polarizations yield subleading logs in the collinear analysis.

To summarize, the longitudinal polarizations yield: (a) the (scalar) helicity flipping contribution (B.12), due to current non conservation, which in broken gauge theories yields the dominant contribution, the probability density being of order  $(mass)^2/Q^2|\Delta|^2$ ; and (b) the helicity conserving contribution (B.15), the only one contributing in the unbroken case, whose probability density is of order  $1/|\Delta|^4$ . The corresponding splitting functions are given in Table 2.

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Table 1

<i>process</i>	$\pi g_H^2$	$P(z)$	$\frac{h^{pole}(\gamma)}{coupling}$	$\frac{h^{pole}(1/2)}{h(0)}$	$\frac{h(1/2)}{h(0)}$
$Zg \rightarrow Q\bar{Q}$	$\frac{\pi 4m^2 C_A^2}{M_Z^2}$	1	$\frac{\alpha_s T_R}{\pi} \frac{1}{(1-\gamma)^2}$	2.00	2.47
$Wg \rightarrow t\bar{b}$	$\frac{\pi m_t^2}{M_W^2} [(1+r)^2 C_A^2 + (1-r)^2 C_V^2]$	1	$\frac{\alpha_s T_R}{\pi} \frac{1}{(1-\gamma)^2}$	1.33	1.49
$\gamma_T g \rightarrow Q\bar{Q}$	$\pi e^2$	$z^2 + (1-z)^2$	$\frac{\alpha_s T_R}{\pi} \frac{2}{3} \frac{1}{(1-\gamma)^2}$	1.71	2.48
$\gamma_T^* \rightarrow q\bar{q}$	$\pi e^2$	$z^2 + (1-z)^2$	$\frac{\alpha_s T_R N_f}{\pi} \frac{2}{3} \frac{1}{(1-\gamma)^2}$	4.00	4.00
$\gamma_L^* \rightarrow q\bar{q}$	$\pi e^2$	$4z(1-z)$	$\frac{\alpha_s T_R N_f}{\pi} \frac{2}{3} \frac{1}{1-\gamma}$	1.00	1.45

Table 2

i	$P^i(z)$	$ coupling\ constant ^2$
S	1	$C_S^2 + C_A^2$
L <sup>1</sup>	$4z(1-z)$	$C_V^2 + C_A^2$
IN	$(1-2z)^2$	$C_V^2 + C_A^2$
OUT	1	$C_V^2 + C_A^2$

<sup>1</sup>Helicity conserving part only. The helicity flipping contribution is equal to the scalar contribution with  $C_S = (m_1 - m_2)/Q$  and  $C_P = (m_1 + m_2)/Q$ .

<sup>2</sup>Effective splitting function after phase space integration (see App. B).

Table 3

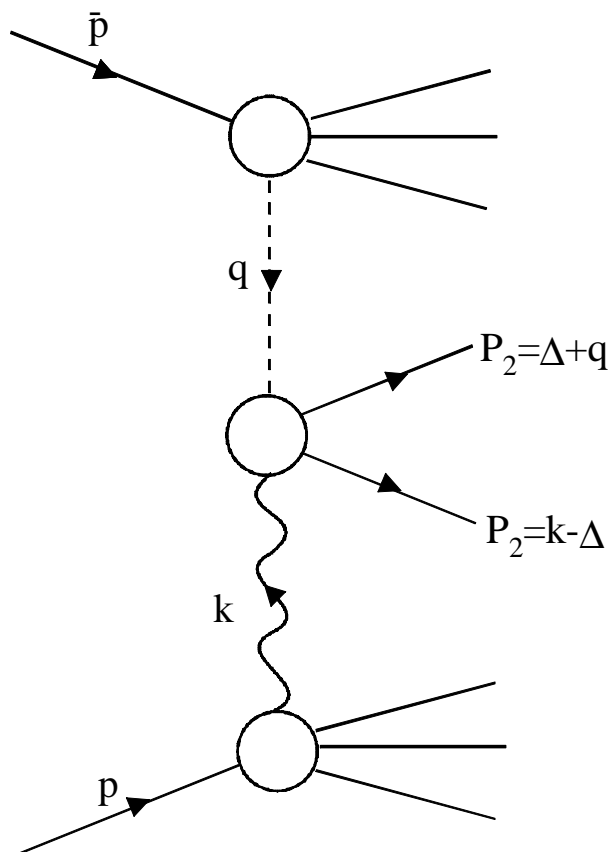
$\lambda$	$\lambda'$	i	$\Gamma_{ab}^i$
+	-	S	$-(C_S - C_P) \frac{\mathbf{k}}{\sqrt{z(1-z)}}$
-	+	S	$(C_S + C_P) \frac{\mathbf{k}}{\sqrt{z(1-z)}}$
+	+	L	$-2(C_V + C_A)Q\sqrt{z(1-z)}$
-	-	L	$2(C_V - C_A)Q\sqrt{z(1-z)}$
+	+	IN	$-(C_V + C_A) \frac{\mathbf{k}}{\sqrt{z(1-z)}}(1-2z)$
-	-	IN	$(C_V - C_A) \frac{\mathbf{k}}{\sqrt{z(1-z)}}(1-2z)$
+	+	OUT	$-i(C_V + C_A) \frac{\mathbf{k}}{\sqrt{z(1-z)}}$
-	-	OUT	$-i(C_V - C_A) \frac{\mathbf{k}}{\sqrt{z(1-z)}}$

## Figure Captions:

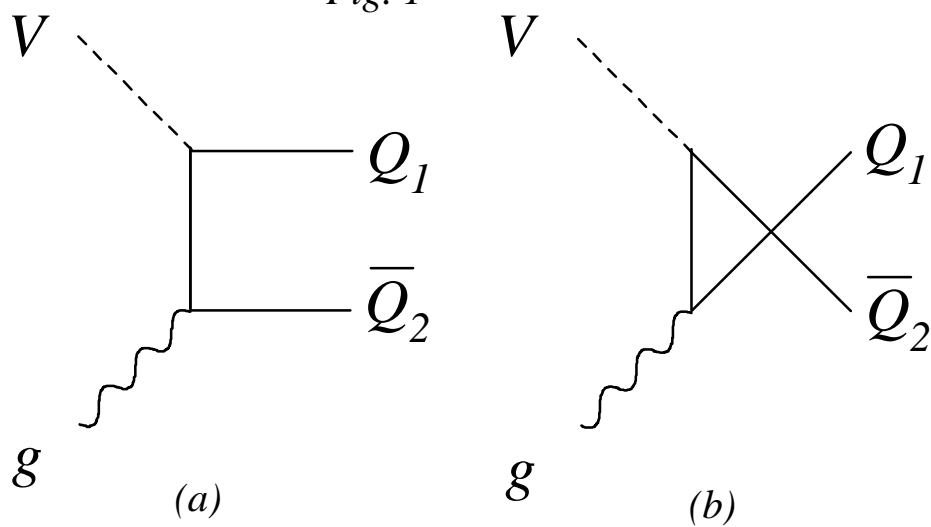
**Fig. 1:** Kinematics of the  $Vg$  contribution to heavy quark production.

**Fig. 2:** The lowest order amplitudes for the hard sub-process  $gV \rightarrow Q_1 \bar{Q}_2$ .

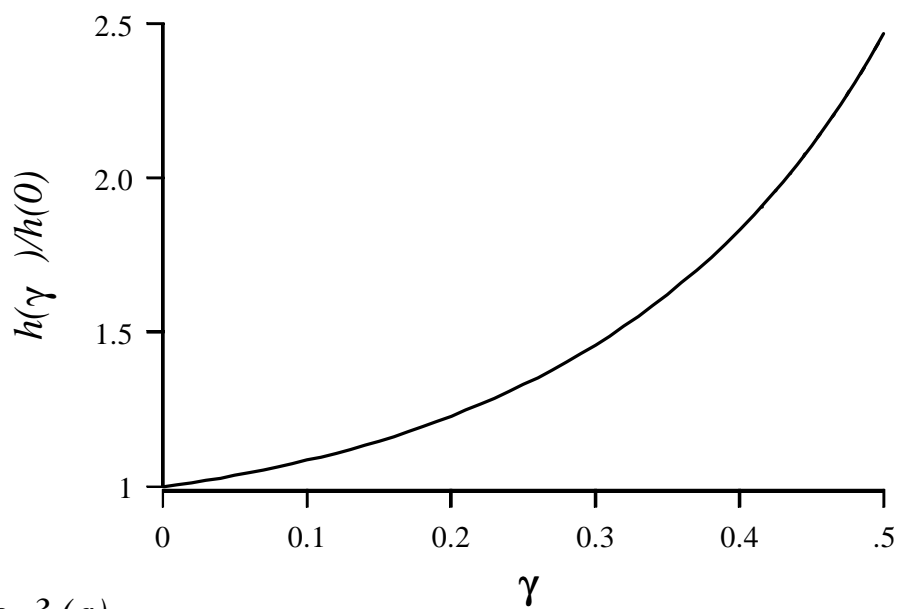
**Fig. 3:** The function  $h(\gamma)$  for (a)  $m_1 = m_2$  and (b)  $m_2/m_1 \simeq 0.03$ , relevant for single top production.



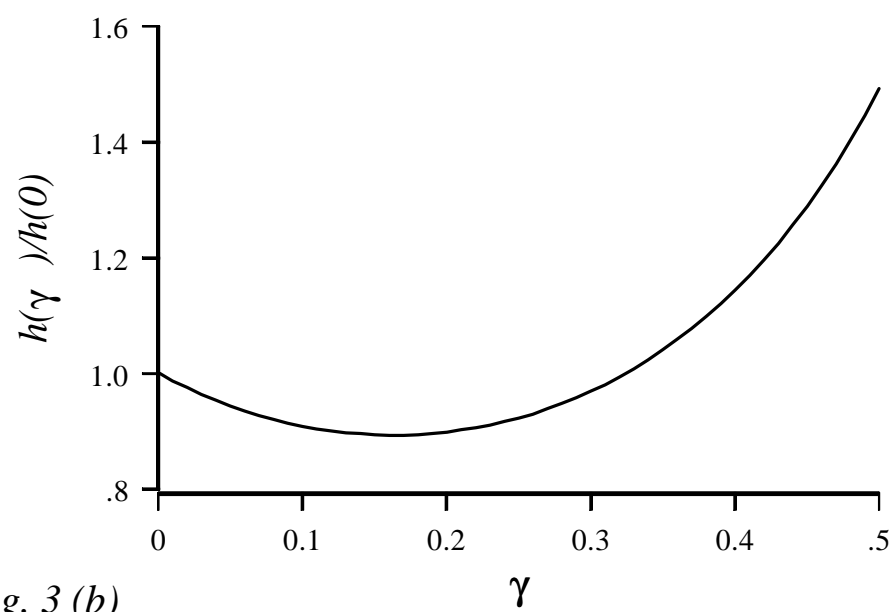
*Fig. 1*



*Fig. 2*



*Fig. 3 (a)*



*Fig. 3 (b)*